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| For Examiner's Use | |
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| Examiner's Initials | |
| Question | Mark |
| 1 | |
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| TOTAL | |

General Certificate of Education
Advanced Level Examination
June 2011

# Mathematics MM04

## Unit Mechanics 4

## Thursday 23 June 2011 9.00 am to 10.30 am

**For this paper you must have:**
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Time allowed**
- 1 hour 30 minutes

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions in the spaces provided.

**1** Three forces, $-\mathbf{i}+\mathbf{j}$, $4\mathbf{i}-2\mathbf{k}$ and $-3\mathbf{i}-\mathbf{j}+2\mathbf{k}$, act at the points whose coordinates are $(0, 2, 1)$, $(3, -1, 0)$ and $(4, 0, -5)$ respectively.

**(a)** Find the resultant of these three forces. *(1 mark)*

**(b)** Find the moment of these three forces about $(0, 0, 0)$. *(6 marks)*

**(c)** Explain why these three forces are equivalent to a couple. *(2 marks)*

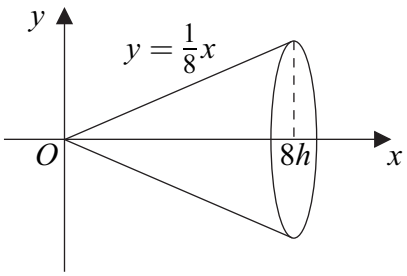
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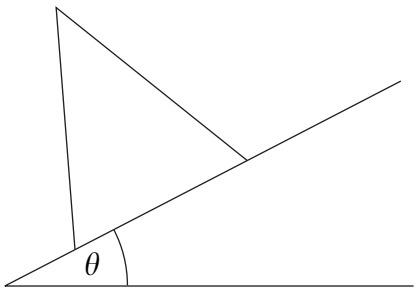
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**2** A uniform solid cone is formed by rotating the finite region bounded by the lines $y = \frac{1}{8}x$, $y = 0$ and $x = 8h$ through $2\pi$ radians about the $x$-axis. The cone is shown in the diagram.




**(a)** Use integration to show that the distance of the centre of mass of the cone from the origin is $6h$. *(5 marks)*

**(b)** The cone rests in equilibrium with its plane face on a rough plane inclined at an angle $\theta$ to the horizontal, as shown in the diagram.




**(i)** On the copy of this diagram **printed on page 6**, show all the forces acting on the cone. *(2 marks)*

**(ii)** If the plane is sufficiently rough to prevent slipping, find the maximum value of $\theta$ for the cone to remain in equilibrium without toppling. *(3 marks)*

**(c)** The cone is now placed with its plane face on another rough inclined plane. The coefficient of friction between the cone and this plane is $\frac{6}{13}$. The angle $\alpha$ between the inclined plane and the horizontal is gradually increased from $\alpha = 0°$.

Determine whether the cone will topple first or slide first. *(4 marks)*

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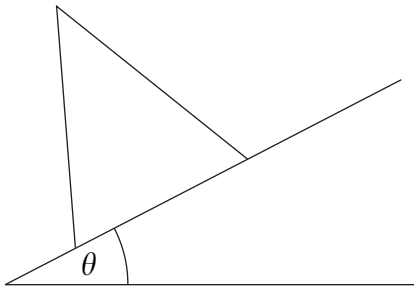
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**(b)(i)**

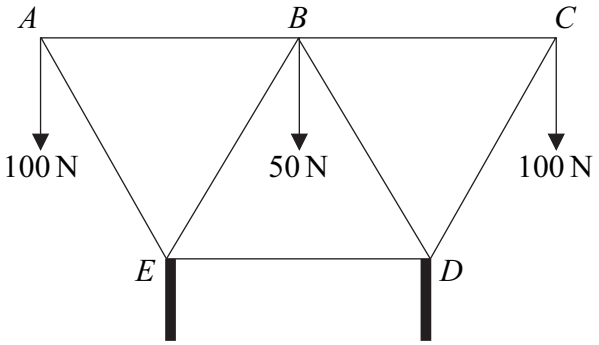


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**3** A framework consists of seven light inextensible smoothly jointed rods, $AB$, $BC$, $CD$, $BD$, $BE$, $AE$ and $ED$. They form three equilateral triangles, $ABE$, $BDE$ and $BCD$. The framework rests in equilibrium in a vertical plane, on smooth vertical supports at $E$ and $D$. Rods $AB$, $BC$ and $ED$ are horizontal, and rod $ED$ is below $ABC$. Loads of 100 N are attached to $A$ and $C$, and a load of 50 N is attached to $B$, as shown in the diagram.




**(a)** Explain why the reactions on the framework at $E$ and $D$ are equal. *(1 mark)*

**(b)** Hence find the magnitude of the reactions on the framework at $E$ and $D$. *(2 marks)*

**(c)** Show that the magnitudes of the forces in the rods $AE$ and $AB$ are 115 N and 57.7 N respectively, correct to three significant figures. *(4 marks)*

**(d)** Find the magnitude of the force in rod $BE$. *(3 marks)*

**(e)** Find the magnitude of the force in rod $ED$. *(3 marks)*

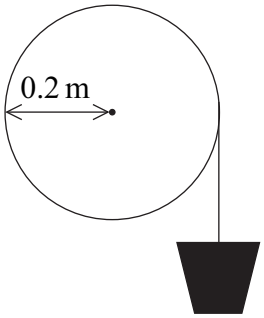
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**4** A light inextensible rope is wrapped several times around a pulley of radius 0.2 m and mass 1.5 kg. A bucket of water of mass 1 kg is attached to the free end of the rope, as shown in the diagram.




The pulley is free to rotate about a smooth fixed horizontal axis through its centre, perpendicular to its plane face.

The system is released from rest with the bucket hanging freely. Assume that the pulley can be modelled as a uniform disc and the bucket as a particle.

**(a)** Show that the moment of inertia of the pulley about the axis is $0.03\,\text{kg}\,\text{m}^2$.

*(2 marks)*

**(b)** Shortly after being released, the bucket is moving with a speed of $2\,\text{m}\,\text{s}^{-1}$ and has fallen a vertical distance of $d$ metres.

**(i)** Calculate the angular velocity of the pulley at this instant. *(2 marks)*

**(ii)** Show that $d = \frac{5}{14}$. *(4 marks)*

**(iii)** Find the tension in the rope. *(5 marks)*

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**5** The forces $\begin{bmatrix} a \\ -3 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 2a \end{bmatrix}$ and $\begin{bmatrix} -8 \\ 0 \end{bmatrix}$ act in the $x$–$y$ plane at the points with coordinates $(2, 6)$, $(3, -5)$ and $(-1, 2)$ respectively.

**(a)** In the case where this system of forces is equivalent to a single force acting at the origin $O$ and a couple $C$, find the magnitude of $C$ and deduce that it is independent of $a$. *(5 marks)*

**(b)** In the case where $a = 4$, this system of forces is equivalent to a single force **F** acting at the point $(d, 0)$.

**(i)** Find **F**. *(2 marks)*

**(ii)** Find $d$. *(2 marks)*

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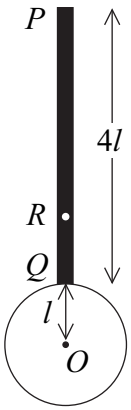
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**6 (a)** Prove, by integration, that the moment of inertia of a uniform solid sphere, of mass $m$ and radius $r$, about a diameter is $\frac{2}{5}mr^2$. *(6 marks)*

**(b)** A pendulum consists of two parts. The upper part can be modelled as a uniform rod $PQ$ of length $4l$ and mass $3m$. The lower part can be modelled as a uniform solid sphere of radius $l$, mass $5m$ and centre $O$. The end $Q$ of the rod is rigidly attached to a point on the surface of the sphere so that $PQO$ is a straight line. The rod is smoothly pivoted at $P$ so that the pendulum is free to swing about a horizontal axis through $P$. The point $R$ is at a distance $l$ above $Q$.

Initially, the pendulum is stationary with $O$ vertically below $P$, as shown in the diagram.




**(i)** Write down the moment of inertia of the rod $PQ$ about a horizontal axis through $P$. *(1 mark)*

**(ii)** Show that the moment of inertia of the pendulum about a horizontal axis through $P$ is $143ml^2$. *(4 marks)*

**(iii)** The pendulum, which is initially at rest, is struck at the point $R$ by a small lump of clay, of mass $m$, moving with speed $v$ horizontally at right angles to the axis through $P$.

Find an expression, in terms of $v$ and $l$, for the angular velocity of the pendulum immediately after the collision. You may assume that the clay sticks to the pendulum throughout the motion. *(6 marks)*

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**END OF QUESTIONS**

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